

### **Ver 21.00 – Base Motion 2/17/2020**

A steady state harmonic base excitation (motion) can be specified in this Ver 21.00. The bearings with station  $J = 0$ , with the exception of Floating Ring Bearing (see example 5 for more details), are connected to the base and the flexible supports with the non-zero stiffness and damping are also considered to be connected to the base, as shown in the Figures 1 and 2, and all the stations connected to the base are subject to the base excitation if specified. i.e., all the rotor/support stations connected to the ground are now considered to be connected to the base if base excitation is present. The foundation is neglected in the base motion analysis.



**Fig. 1 – Single Base** 



**Fig. 2 – Multiple Bases** 

For the steady state harmonic base excitation analysis, the system must be linear and bearings are linear bearings. This is a linear analysis.

The base motion inputs are entered in the Base Motion tab under Model – Data Editor, as shown in Figure 3. The inputs are described below:



**Fig. 3 – Inputs for Base Motion (Single Base)**

1) Base Type: The Base Type can be Single, as shown in Fig. 1, or Multiple as shown in Fig. 2.



For a Single Base model, all the stations connected to the same base are subject to the same base motion. For a Multiple Bases model, the stations connected to the different base can have different base motion in amplitude and phase, but the base excitation frequency is the same for all the bases, only the amplitude and phase can be different. Each base has 4 degrees-of-freedom, as described in the lateral vibration model, i.e., two translations  $(X, Y)$  and two rations (Theta-X and Theta-Y). For a Single Base, the inputs are illustrated in Fig. 3. Fig. 4 shows a multiple bases model input. There is a "station reminder" automatically shown above the motion input. It shows the number of stations connected to the base and the station numbers. For a multiple bases model, the base motion for all the connected stations must be entered.

			Rotor Bearing System Data									×
Units/Description   Material   Shaft Elements   Disks Unbalance Bearings   Supports   Foundation User's Elements Shaft Bow   Time Forcing   Harmonics   Base Motion   Torsional/Axial Axial Forces   Static Loads   Constraints   Misalignments												
	Base Type: Multiple ▼											
	Steady State Base Harmonic Motion: Excitation Frequency is a function of Rotor Speed or a Constant											
	Excitation Frequency (cpm = wo + w1 * rpm + w2 * rpm $2$ )											
			Wo: 6000		W1: 0			0 W2:				
	Amplitude Multiplier (A = Ao + A1 * rpm + A2 * rpm^2)											
	Ao: 1				A1: $ 0$ A2: 0							
Steady State Harmonic Base Motion: $q = A^*$ [qc * cos (wexc*t) + qs * sin(wexc*t) ] (gc,gs) are the displacement amplitudes in (cos,sin) components Excitation frequency wexc (rad/sec) = cpm $*(2^{\circ}pi/60)$ . pm = reference shaft speed, rotor 1 speed												
	3 Stations connected to the Base: 1, 6, 7											
		stn l	Xc-cos	Xs-sin	Ye-cos	Ys-sin	ThetaXc	Theta <sup>×s</sup>	ThetaYc	ThetaYs	Comments	
	1	1	0.1	0.	0.	0.2	0	0	0	0	For Multiple Bases	
	2	6	0.05	0	0	0.1	0.	0.	0.	0		
	3	7	0.03	0	0.	0.05	0.	0	0.	0.		
	4 5											
	6											
<b>Insert Row</b> Delete Row Unit:(1) - Amplitude: inch, radian												
Tor K Save Save As Close Help												

**Fig. 4 – Inputs for Base Motion (Multiple Bases)**

Note that if the rotational displacements are specified in the base motion, the bearings connected to the base must have rotational stiffness and/or damping to transmit the base motion.

2) Excitation Frequency: The base motion frequency (excitation frequency) can be either a function of rotor speed or a constant frequency, or the excitation frequency varies at a constant rotor speed.



If the excitation frequency is a function of rotor speed or a constant, the analysis is performed for a range of rotor speed, as illustrated in Fig. 5. If the excitation frequency varies at a constant rotor speed, the analysis is performed for a range of excitation frequency, as illustrated in Fig. 6.



**Fig. 5 – Excitation frequency is a constant**



#### **Fig. 6 – Excitation frequency varies at a constant rotor speed**

3) Base Motion: The base motion is described as a steady state harmonic motion.

For the *i*<sup>th</sup> base: 
$$
z_i = A \times \left[ z_{ci} \cos \left( \omega_{ex} t \right) + z_{si} \sin \left( \omega_{ex} t \right) \right] = A \times \left| z \right| \cos \left( \omega_{ex} t - \phi \right)
$$
(1)

Note that the displacement expression uses a phase lag  $(-\phi)$ , and the force expression uses a phase lead.

where *A* is a speed dependent amplitude multiple. In general, the base motion is speed independent, therefore  $A_0=1$ ,  $A_1=A_2=0$ . For a multiple base model, the base excitation frequency is the same, but the amplitude and phase can be different by specifying different cosine and sine components ( $z_c$  and  $z_s$ ) of the motion. For every base motion, 4 degrees-of-freedom can be specified: two translations  $(x,y)$  and two rotations  $(\theta_x, \theta_y)$ . Since the motion is transmitted through bearings to the rotor system. If rotational base motion is specified  $(\theta_x, \theta_y)$ , then the bearing connected to this base must have the rotational stiffness and/or damping to transmit this base motion to the rotor system. Otherwise, the rotational base motion will be ignored.

4) Analysis: When performing the base motion analysis, analysis option 23, the rotor speed input depends on the excitation frequency type entered in the Base Motion Input. As said before, if the excitation frequency is a function of rotor speed or a constant, the analysis can be performed in a range of rotor speed, and if the excitation frequency varies at a constant rotor speed, the analysis is performed at a constant rotor speed, as illustrated in Fig. 7.



**Fig. 7 – Rotor speed input for the base motion analysis**

The results can be viewed from the Postprocessor in both text and graphic formats. Several examples are employed to illustrate the use of Base Motion Analysis. Mathematically, the absolute displacement of the rotor due to the base motion can be verified using the Steady State harmonic Excitation Analysis. It will be demonstrated in the following examples.



# **Example 1 – Single Degree-of-Freedom**

For a single DOF system, a mass *m* is supported by a spring *k* and a damping *c*. The spring and damping connected to the base are subject to a base motion  $z(t)$ .



The equation of motion of the mass can be obtained by applying the Newton's  $2<sup>nd</sup>$  law for the absolute displacement *x* is:

$$
m\ddot{x} = F_k + F_c \tag{2}
$$

where  $F_k$  and  $F_c$  are the spring and damping forces acting on the mass  $m$ .

$$
F_k = -k(x-z) \tag{3}
$$

$$
F_c = -c(\dot{x} - \dot{z})\tag{4}
$$

i.e.,

$$
m\ddot{x} = -k(x-z) - c(\dot{x}-\dot{z})
$$
\n(5)

or

$$
m\ddot{x} + c\dot{x} + kx = kz + c\dot{z}
$$
 (6)

For a harmonic base motion

$$
z = z_c \cos(\omega_{exc}t) + z_s \sin(\omega_{exc}t)
$$
\n(7)

Therefore, the equation of motion becomes:

$$
m\ddot{x} + c\dot{x} + kx = (kz_c + c\omega_{ex}z_s)\cos\omega_{ex}t + (kz_s - c\omega_{ex}z_c)\sin\omega_{ex}t
$$
\n(8)

Define the relative displacements with respect to the base motion *z*:

$$
u = x - z \qquad \Rightarrow \qquad x = u + z \tag{9}
$$

Substitution of Eq. (9) into Eq. (6), the equation of motion in the relative displacement form:  $F_k = -k(x - z)$ <br>  $F_c = -c(\dot{x} - \dot{z})$ <br>  $m\ddot{x} = -k(x - z) - c$ <br>  $m\ddot{x} + c\dot{x} + kx = kz + c$ <br>
rmonic base motion<br>  $z = z_c \cos(\omega_{ext}) + z$ <br>
re, the equation of r<br>  $m\ddot{x} + c\dot{x} + kx = (kz_c)$ <br>
the relative displace<br>  $u = x - z \implies x$ <br>
tion of Eq. (9) into<br>  $m\dd$ 

$$
m\ddot{u} + c\dot{u} + ku = -m\ddot{z}
$$
 (10)

#### **Case 1: BaseMotion\_1a.rot**

The first case is taken from "Applied Mechanical Vibrations" by David V. Hutton, page 84.

 $m = 8$  Lb,  $k = 40$  Lb/in,  $c = 0$ ,  $z_c = 0$ ,  $z_s = 0.2$  in, and  $\omega_{\text{exc}}$  = 115 Hz (6900 cpm = 722.57 rad/sec)

The system undamped natural frequency is:

$$
z = z_c \cos(\omega_{exc}t) + z_s \sin(\omega_{exc}t)
$$
  
re, the equation of motion becomes:  

$$
m\ddot{x} + c\dot{x} + kx = (kz_c + c\omega_{exc}z_s)\cos\omega_{exc}t + (kz_s - c\omega_{exc}z_c)
$$
  
the relative displacements with respect to the base motif  
 $u = x - z \implies x = u + z$   
tion of Eq. (9) into Eq. (6), the equation of motion i  

$$
m\ddot{u} + c\dot{u} + ku = -m\ddot{z}
$$
  
**BaseMotion\_1a.rot**  

$$
t \cose is taken from "Applied Mechanical Vibrations" $m = 8$  Lb,  $k = 40$  Lb/in,  $c = 0$ ,  $z_c = 0$ ,  $z_s = 0.2$  in, and  
 $\omega_{exc} = 115$  Hz (6900 cpm = 722.57 rad/sec)  
term undamped natural frequency is:  

$$
\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/386.088}} = 43.94
$$
 rad/sec = 419.57 cpm  
quency ratio is defined as below  

$$
\gamma = \frac{\omega_{exc}}{m} = 16.45
$$
$$

The frequency ratio is defined as below

$$
\gamma = \frac{\omega_{\text{exc}}}{\omega_n} = 16.45
$$

For this high frequency ratio, i.e., the excitation frequency is much higher than the system natural frequency, the inertia of the mass keeps it from moving much, so that the relative motion consists primarily of the base motion relative to the mass. The mass steady state vibration amplitude is 0.00074 in. The mass relative displacement to the base is 0.20074 in.

The related rotor-bearing model and base motion inputs and analysis inputs are shown below:









**Fig. 9 – Input data for Example 1 Case 1 (BaseMotion\_1a.rot)**

The constraints are used to limit the system to be a single degree-of-freedom in the X direction only for the purposes of comparison and understanding the base motion.

The reason to run the analysis from 0 to 100 rpm with an increment of 10 rpm is primarily for the easy presentation of the results. For this simple system, all the system parameters are not dependent on the rotor speed, therefore the results are also independent of the speed.

The results, the absolute displacement and the relative displacement of the mass and the force transmitted to the base, from the Base Motion Analysis are shown below:







**Fig. 10 – Outputs for Example 1 Case 1 (BaseMotion\_1a.rot)**

As said that the absolute displacement can be verified by using the Steady State Harmonic Excitation Analysis according to Eq. (8). The steady state harmonic excitation is a function of bearing coefficients ( $k$  and  $c$ ) and base motion ( $\zeta$  and  $\omega_{\text{exc}}$ ). However, the force transmitted to the base through the bearing cannot be verified using this analysis, since the relative displacement is not obtainable from the Steady State Harmonic Excitation Analysis without knowing the base motion.

The bearing transmitted force from Base Motion is:

$$
F = k(x-z) + c(\dot{x} - \dot{z})\tag{11}
$$

The bearing transmitted force from the Steady State Harmonic Excitation is:

$$
F = kx + c\dot{x} \tag{12}
$$

The input and output for the steady state harmonic analysis are show below. Note that the force expression uses a phase lead  $(\phi)$  and the displacement uses a phase lag.







**Fig. 10 – Verification of Base Motion using the Steady State Harmonic Excitation Analysis**

#### **Case 2: BaseMotion\_1b.rot**

Since this simple system has a constant and speed independent natural frequency, let us consider a wide range of excitation frequency at a constant rotor speed to plot the response versus the frequency ratio. To limit the response amplitude at the resonance, have:

frequency ratio =1, let us add a damping coefficients of 
$$
c = 0.5
$$
 in the bearing. Then we have:  
undamped natural frequency =  $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/386.088}} = 43.94$  rad/sec = 419.57 rpm

damping factor = 
$$
\zeta = \frac{c}{2m\omega_n} = \frac{0.5}{2 \times \frac{8}{386.088} \times 43.94} = 0.275
$$

2 damped natural frequency =  $\omega_a = \omega_n \sqrt{1 - \zeta^2} = 42.25$  rad/sec=403 cpm



The relevant inputs are shown in Fig. 11 below:





**Fig. 11 – Example 1 Case 2: Excitation frequency from 5 to 1000 cpm**

The absolute and relative displacements of the mass and the force transmitted to the base versus the frequency ratio are shown in Fig. 12 below:





 $0<sup>1</sup>$  $\mathbf{0}$ Excitation Frequency (cpm)

**Fig. 12 – Base Motion Results for Example 1 Case 2**

To change the graphic scales and headings, go to Options – Settings and macke necessary changes. To scale the X-axis from Excitation Frequency to Frequency Ratio (Exec Freq/natural Freq), multiplying a scale factor (1/Natural Freq  $= 1/419.57=0.00238$ ) as shown below.



As expected, the absolute displacement starts with the base motion at  $\gamma = 0$  and reaches the maximum absolute displacement before the resonance  $\gamma =1$ , and the absolute displacement equals to the base motion of 0.2 inches at  $\gamma = 0$ , and  $\sqrt{2}$ . The absolute displacement decreases as the frequency ratio increases after the resonance. The relative displacement starts from zero and reaches the maximum relative displacement after the resonance and approaches to the base motion when the frequency ratio is very high. At very low frequency ratio, i.e., the excitation frequency is much less than the system natural frequency, the mass vibrates with the base and there is little relative motion between the mass and the base. At very high frequency ratio, the mass is nearly stationary and the relative motion is primary the base motion.

Again, the maximum absolute displacement, 0.4253 inches at 395 rpm can be verified by using the steady state harmonic excitation analysis. The excitation frequency is 395 cpm (41.36 rad/sec) and the excitation force, according to Eq. (8), is:

$$
kz + cz = k (z_c \cos \omega_{ex}t + z_s \sin \omega_{ex}t) + c\omega_{exc} (z_s \cos \omega_{ex}t - z_c \sin \omega_{ex}t)
$$
  
= 40(0.2 sin  $\omega_{exc}t$ ) + 0.5 ×  $\frac{395 \times 2\pi}{60}$  (0.2 cos  $\omega_{exc}t$ )  
= 8 sin  $\omega_{exc}t$  + 4.13643 cos  $\omega_{exc}t$  = 9.00611 cos  $(\omega_{exc}t - 62.659^\circ)$   
= 9.00611 cos  $(\omega_{exc}t + 297.341^\circ)$ 

The input and output are shown below:





**Fig. 13 – Steady State Harmonic Excitation at 395 cpm for Example 1 case 2**

#### **Example 2 - 2 Degrees-of-Freedom**

The second example is a 2 DOF system, as shown below. For more details, see "Structural Dynamics" by Roy R. Craig, Jr., page 240.



 $m_1=8$  lb,  $k_1 = 40$  lb/in,  $c_1 = 0.2$  lb-sec/in  $m_2$ =15 lb,  $k_2$  = 100 lb/in,  $c_2$  = 0.1 lb-sec/in base motion:  $z_c = 0$ ,  $z_s = 0.2$ ,  $\omega_{\text{exc}}$  from 5 cpm to 1000 cpm

**Base Motion** 

#### **Fig. 14 – Two Degrees-of-Freedom System**

The equation of motion:

The equation of motion:  
\n
$$
\begin{bmatrix} m_1 & 0 \ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & (c_1 + c_2) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1 + k_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} 0 \\ k_2 z + c_2 \dot{z} \end{Bmatrix}
$$
\n(13)

Define the relative displacements with respect to the base motion *z*:

$$
\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} x_1 - z \\ x_2 - z \end{Bmatrix} \implies \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{Bmatrix} z \\ z \end{Bmatrix}
$$
 (14)

Substitution of Eq. (14) into Eq. (13), the equation of motion in the relative displacement<br>form:<br> $\begin{bmatrix} m_1 & 0 \ 0 & \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \vdots \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \ c_2 & (a + c_1) \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \vdots \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \ k_2 & (b$ form:

form:  
\n
$$
\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & (c_1 + c_2) \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1 + k_2) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = - \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{z} \end{bmatrix} (15)
$$

The response can be solved in either coordinate system (absolute or relative displacements). For a single base model, it is convenient to solve the equation of motion in the relative displacement, Eq. (15). However, for a multiple bases model, will be presented later, the absolute displacement, Eq. (13) will be used.

Again, this is also a simple model and the system parameters are independent from the rotor speed. The system natural frequencies are constant and not varied with the rotor speed.

The system natural frequencies and damping factors can be obtained from the whirl speed analysis:



Undamped Natural Frequencies:

 $\omega_{n1} = 33.58 \text{ rad/sec} = 320.67 \text{ cpm}$  $\omega_{n2} = 66.38 \text{ rad/sec} = 633.8 \text{ cpm}$ 

Damped Natural Frequencies and Damping Factors:









# **Case 1: BaseMotion\_2a.rot**

The relevant inputs are shown below in Fig. 15. Note that in this model, the support stiffness and damping are entered as a bearing.









**Fig. 15 – Inputs for Example 2**





Mass  $m_1$  absolute and relative displacements (station 1)



Fig. 16 – **Displacements for Mass**  $m_1$ 



Mass  $m_2$  absolute and relative displacements (station 3)





The transmitted forces for bearing 1 and bearing 2 are shown below. The force transmitted through bearing 2 is acting on the base.



**Fig. 18 – Forces Transmitted through Bearings**

Again, the absolute displacements can be verified by using the steady state harmonic

excitation analysis. The excitation acting on the mass 
$$
m_2
$$
 (station 3) is:  
\n
$$
kz + cz = k(z_c \cos \omega_{exc}t + z_s \sin \omega_{exc}t) + c\omega_{exc}(z_s \cos \omega_{exc}t - z_c \sin \omega_{exc}t)
$$
\n
$$
= (kz_c + c\omega_{exc}z_s) \cos \omega_{exc}t + (kz_s - c\omega_{exc}z_c) \sin \omega_{exc}t
$$

For the excitation frequency of 320 cpm, the harmonic excitation is:

$$
\left(0.1 \times \frac{320 \times 2\pi}{60} \times 0.2\right) \cos \omega_{\text{exc}} t + (100 \times 0.2) \sin \omega_{\text{exc}} t
$$

The harmonic excitation input:



**Fig. 19 – Harmonic Excitation at** *m***<sup>2</sup> (station 3)**

The results for the excitation frequency at 320 cpm can be verified. In order to shown the plots, we ran the steady state harmonic analysis for a speed range of 0-1000 rpm. The displacement results are:





**Fig. 20 – Responses for the Harmonic Excitation**

# **Case 2: BaseMotion\_2b.rot**

In this model, we can also move the bearing 2 data into the support tab, as shown in Fig. 21. The results are identical to the previous model and not repeated here.



**Fig. 21 – Alternative inputs for Bearing 2**

### **Example 3 – Multiple Degrees-of-Freedom**

A multiple DOF system, as shown in Fig. 22, is used in this example. At the operating speed of 3,000 rpm, the first five natural frequencies are: 4,312 (backward), 4,358 (forward), 7,342 (backward), 7,619 (forward), and 100,684 (backward) rpm and their associated whirling modes are shown in Fig. 23.



**Fig. 22 – Example 3**







**Fig. 23 – The first five modes at 3,000 rpm**

## **Case 1: BaseMotion\_3a.rot**

A single base is assumed in this case. The base has a motion in Y direction and the excitation frequency is from 200 to 10,000 cpm with an increment of 50 cpm. This excitation frequency will excite the first two system natural frequencies and far below the third natural frequency. The base motion and the analysis input are shown below:





**Fig. 24 – The Base Motion and Analysis Input**

The displacements (absolute and relative) at the disk (station 1) and bearings (stations 3 and 7) are shown in Fig. 25 below:












**Fig. 25 – The Results for Base Motion**



Forces transmitted through bearings to the base are:

**Fig. 26 – The Force Transmitted to the Base**

Although the bearings are isotropic and uncoupled in the X and Y directions in this example, as shown below, the  $\overrightarrow{X}$  and  $\overrightarrow{Y}$  motions are coupled through the gyroscopic effect. So, even with the base motion in the Y direction only, the responses occur at both X and Y directions with dominant Y displacement.



Again, the absolute displacements can be verified using the Steady State Harmonic Excitation Analysis as described before. For comparison purposes, the excitation frequency at 4,350 cpm is selected. The excitation forces can then be calculated and entered in the harmonic excitation input, as shown below. The analysis is run at a constant rotor speed of 3,000 rpm. The results are also listed below.





**Fig. 27 – The Steady State Harmonic Excitation**

**\*\*\*\*\*\*\*\*\*\*\*\*\*\* Harmonic Response due to Shaft (1) Excitation \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*** \*\*\* Excitation Frequency = 4350.0



The absolute displacements are identical to the results from the base motion analysis. However, the forces transmitted through bearings are not the same since the steady state harmonic excitation analysis did not include the base motion.

### **Case 2: BaseMotion\_3b.rot**

In this case, the Single Base is replaced by Multiple Base Option with the identical base motion, as shown in Fig. 28.



**Fig. 28 – Multiple Base**

The results are identical to the Case 1 and not presented here. However, when plotting the relative displacements, one must select which base will be used for the relative displacement as shown in Fig. 29.



**Fig. 29 – The Relative Displacement**

### **Case 3: BaseMotion\_3c.rot**

In this case, both bearings are subject to different base motion, as shown in Fig. 30. Note that the base motion can have different amplitude and phase, but the same frequency. The first bearing (station 3) has a base motion in the Y direction only. The second bearing (station  $\overline{7}$ ) has the base motion in both X and Y directions.



**Fig. 30 – The Base Motion**

The absolute displacements for the disk and both bearings are shown in Fig. 31. The relative motions for both bearings are shown in Fig. 32. The forces transmitted through bearings are shown in Fig. 33.







**Fig. 31 – The Absolute Displacements**





**Fig. 32 – The Relative Displacements**





**Fig. 33 – The Bearing Transmitted Forces**

To verify the absolute displacements caused by the base motion, the steady state harmonic excitation analysis is again used. For comparison purposes, the frequency at 4,350 cpm is selected. The amplitudes and phases are entered accordingly as shown in Fig. 34.

The right hand side of the equation for the Steady State Harmonic Excitation:

$$
\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} z_x \\ z_y \end{Bmatrix} + \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{z}_x \\ \dot{z}_y \end{Bmatrix}
$$
(13)

where

$$
\begin{aligned}\n\begin{bmatrix}\nz_x \\
z_y\n\end{bmatrix} &= \begin{Bmatrix}\nx_c \\
y_c\n\end{Bmatrix} \cos(\omega_{exc}t) + \begin{Bmatrix}\nx_s \\
y_s\n\end{Bmatrix} \sin(\omega_{exc}t) \\
\begin{Bmatrix}\n\dot{z}_x \\
\dot{z}_y\n\end{Bmatrix} &= \omega_{exc} \left( \begin{Bmatrix}\nx_s \\
y_s\n\end{Bmatrix} \cos(\omega_{exc}t) - \begin{Bmatrix}\nx_c \\
y_c\n\end{Bmatrix} \sin(\omega_{exc}t) \right)\n\end{aligned}
$$

and

$$
\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} F_{xc} \\ F_{xc} \end{Bmatrix} \cos(\omega_{exc}t) + \begin{Bmatrix} F_{xs} \\ F_{ys} \end{Bmatrix} \sin(\omega_{exc}t)
$$



**Fig. 34 – The Steady State Harmonic Excitation**

Since the comparison is selected only in one frequency, the results are compared as below:



## **Whirling Direction**

To fully understand the effect of the base motion, we need to also examine the precessions of the base motion and the rotor motion. In this Case 3, the base motion at station 3 is a straight line motion which can excite both forward and backward whirling modes. The base motion at station 7 is a backward precessional elliptical motion which tends to excite the backward precessional modes more. To view the base motion, there is a Tool in DyRoBeS which can help you visualize the base motion, as shown below:



## Base Motion at Station 3:



#### Base Motion at Station 7:



Use the Animation Play option, you can visualize the motion better. Now, let us go back to examine the entire rotor response (absolute displacements) from the Base Motion Analysis at the excitation frequency of 200, 4312, 4358, 7342, and 7619 cpm.





In the graph, the properties of the max orbit are printed. The value a is the semi-major axis, and b is the semi-minor axis. The positive sign of the semi-minor axis indicates the orbit is a forward precession and negative sign indicates the backward precession. The **red arrow** represents the base motion. At very low excitation frequency (I.e., low frequency ratio), the rotor moves with the base and there is no relative movement between rotor and base. The rotor whirls in the forward precession for the stations 1 (max orbit) and 2. The station 3 is a straight line motion which is the same as the base motion. After the straight line motion, the rest of the rotor whirls in the backward precession, same as the station 7 base motion. So, the rotor is whirling in a so-called mixed precession.

At the excitation frequency of 4350 cpm, the entire rotor whirls in the backward precession at resonance. The rotor deflection shape is similar to the first mode. The rotor response is far larger than the base motion at or near resonance.



At the excitation frequency of 7500 cpm, the entire rotor whirls also in the backward precession. The rotor deflection shape is similar to the third mode.



To check the whirling direction for all the excitation frequency range, the easiest way is to plot the elliptical orbital axes (semi-major and semi-minor axes). The negative semiminor axis indicates a backward whirl.

The orbital axes for the stations 1 and 7 are shown below. It shows that the station 1 starts with forward precessions at low excitation frequency and becomes backward precessions around 3400 cpm, then turns to forward precessions again after 8550 cpm. However, for station 7, it starts with backward precessions, and becomes forward precessions only between 4600 cpm and 4800 cpm.





## **Case 4: BaseMotion\_3d.rot**

In this case, change the base motion  $x_s$  from 0.05 to -0.05 at station 7. This will change the base motion at station 7 from the previous backward precession to forward precession. The rests of data are unchanged.



Now, at 200 cpm, again, at such low excitation frequency and low frequency ratio, the rotor moves with the base. It is still a mixed precession, however, the stations 1 and 2 before the bearing #1 (station 3) are backward precessions now, and after the straight line motion at station 3, the rotor whirls in the forward precession same as the station 7 base motion.

At 4350 cpm, the response is at and near resonance. The entire rotor whirls in the forward precession and the rotor response is far larger than the base motion. The rotor deflection shape is similar to the second mode. At 7500 cpm, the rotor whirls in the forward precession. The rotor deflection shape is similar to the fourth mode.





# **Case 5: BaseMotion\_3e.rot**

Let us consider the base motion is a purely forward circular motion as shown below:



Now, you already know the results. Yes, this base motion will only excite the forward modes and no backward precession will be present. Since the system is an isotropic system, the rotor response orbits will be purely forward circular orbits.











# **Case 6: BaseMotion\_3f.rot**

Consider a purely backward circular base motion in this example.



Results are as expected.







#### **Example 4 – Industrial Compressor (BaseMotion\_4a.rot)**

An industrial compressor, as shown below, is used in this example. The rotor assembly is supported by two bearings at stations 2 and 4. The compressor design speed is 35,000 rpm. The bearing  $#1$  at station 2 is a 3-lobe bearing and the bearing  $#2$  at station 4 is a tilting pad bearing. Bearing Type 15 is used in this example.



**Fig. 35 – An Industrial Compressor**





At 35,000 rpm, the first 4 precessional modes are shown in Fig. 36. It shows that the base motion will most likely excite the  $1<sup>st</sup>$  and  $2<sup>nd</sup>$  modes.



**Fig. 36 – The first 4 Precessional Modes**

### **Case 1: BaseMotion\_4a.rot**

For verification purposes, only the  $2<sup>nd</sup>$  bearing at station 4 is subject to a base motion and the  $1<sup>st</sup>$  bearing at station 2 is not subject to any base motion. Therefore, two bases are utilized in this model. The base 1 is not moving and the base 2 has a harmonic motion in Y direction. The base motion is shown in Fig. 37. The base excitation only at station 4 (base 2) varies from 1000 cpm to 50,000 cpm with an increment of 500 cpm at a constant rotor speed of 35,000 rpm.



**Fig. 37 – Base Motion**

The absolute displacements at both bearings are shown in Fig. 38. The relative displacements are shown in Fig. 39. The bearing transmitted forces are shown in Fig. 40.





**Fig. 38 – The Absolute Displacements due to Base Motion**





**Fig. 39 – The Relative Displacement due to Base Motion**





**Fig. 40 – The Transmitted Bearing Forces due to Base Motion**

At low excitation frequency, 1000 cpm, again, the rotor moves with the base motion and there is little relative movement between the rotor and base. Also, the straight line base motion can excite both forward and backward precessions. In this case and at this low frequency, the rotor moves nearly straight line motion  $(b=0)$  which is the same as the base motion.

The maximum response occurs at the impeller side (station 7). Again, at low excitation frequency, the rotor moves with the base motion (straight line) and proceeds with a backward whirl, then reaches the resonance at 19,500 cpm. The motion becomes forward precession after 21,500 cpm, This is understandable since the  $1<sup>st</sup>$  mode at 18,233 cpm with a log. Decrement of 2.0667 is a backward mode and the  $2<sup>nd</sup>$  mode at 21,317 cpm with a log. Decrement of 2.4541 is a forward mode.

At station 4 where the base motion occurs, the motion starts from a straight line motion, then backward, and forward, and backward.








The bearing coefficients for the bearing 2 (station 4) at 35,000 rpm are calculated below:



Based on Eq. (13), we can calculate the equivalent harmonic excitation at the frequency of 22,500 cpm as shown in Fig. 41.



**Fig. 41 – The Equivalent Harmonic Excitation at frequency of 22,500 cpm** 

The absolute displacements due to base motion and steady state harmonic excitation at the frequency of 22,500 cpm are identical as expected.





### **Case 2: BaseMotion\_4b.rot**

In this case, both bases are subject to the same base motion as shown in Fig. 42. At rotor speed of 35,000 rpm, the bearing coefficients are:



Again, for the comparison purposes, the equivalent steady state harmonic excitations at the frequency of 22,500 cpm are calculated as shown in Fig. 43. Note that, the base motion at station 2 only has the Y movement, but the steady state excitations exist in both X and Y directions due to the coupled bearing stiffness and damping coefficients at station 2. The absolute response for both base motion and steady state harmonic excitation are listed for comparison.



**Fig. 42 – Base Motion**



**Fig. 43 – Equivalent Harmonic Excitation**

### **\*\*\*\*\*\*\*\*\*\*\*\* Harmonic Response due to Base Motion (Excitation) \*\*\*\*\*\*\*\*\*\*\*\*\* \*\*\* Excitation Frequency = 22500. cpm \*\*\***



\*

### **\*\*\*\*\*\*\*\*\*\*\*\*\*\* Harmonic Response due to Shaft (1) Excitation \*\*\*\*\*\*\*\*\*\*\*\*\*\*\* \*\*\* Excitation Frequency = 22500. cpm \*\*\***





The responses at both bearings for the base motion are shown below;





**Fig. 44 – Absolute and Relative Displacements at Bearings**





**Fig. 45 – The Transmitted Bearing Forces due to Base Motion**

# **Case 3: BaseMotion\_4c.rot**

In Case 2, although Multiple Bases are used, two bases have the same base motion. So, in this case, a Single base model is used as shown in Fig. 46. The results for the base motion in this case are identical to the results in Case 2 and are not repeated here.



**Fig. 46 – Base Motion – Single Base**

# **Case 4: BaseMotion\_4d.rot**

In this case, bearing #2 (station 4) is connected to a support (station 8) as shown in Fig. 47. The base motion is acting on the stations 2 and 8.



**Fig. 47 – System Model**



**Fig. 48 – Base Motion**









# **Fig. 49 – Bearing and support data**

At the rotor speed of 35,000 rpm, the first five natural frequencies and modes are shown in Fig. 50.







**Fig. 50 – The first five natural frequencies and modes**

The absolute and relative displacements at stations 4 and 8, bearing #2 and its support, are shown in Fig. 51.









**Fig. 51 – The Absolute and Relative Displacements at Stations 4 and 8**

Again, let us examine the impeller station (station 7) where the maximum displacement occurs. It starts from with the base motion (a straight line motion), then backward precession when approaching the  $1<sup>st</sup>$  mode (backward mode), after 15,000 cpm, the motion becomes forward and reaches the maximum peak at 16,000 cpm. From the previous Whirl Speed/Stability Analysis (Fig. 50), it shows that the  $2<sup>nd</sup>$  mode (forward mode) has a slightly smaller damping (log. Decrement) than that of the  $1<sup>st</sup>$  mode (backward mode). The second peak occurs around 40,000 cpm and the motion is a backward precession. This can also be observed from the mode frequency and damping at mode #4.



### **Case 5: BaseMotion\_4e.rot**

This case is identical to the previous Case 4, except the base motion has entered as multiple bases. The results are identical to Case 4, and not repeated here.



**Fig. 52 – Base Motion – Multiple Bases**

Let us consider a steady state harmonic excitation with a frequency of 40,000 cpm acting on the station 2 and station 8 at the rotor speed of 35,000 rpm. At 35.000 rpm, the bearing coefficients are:



Rotor Bearing System Data									×
Units/Description		Material		Shaft Elements   Disks Axial Forces   Static Loads   Constraints   Misalignments   Shaft Bow   Time Forcing	Unbalance	Bearings	Supports   Foundation   Harmonics	User's Elements Base Motion   Torsional/Axial	
				Steady State Harmonic Excitation: Excitation Frequency varies at a Constant Rotor Speed					
	Excitation Frequency (cpm)	Start: 40000		Stop: $ 0 $		Increment. 0			
		Ao: $ 1$		Amplitude Multiplier (A = Ao + A1 * rpm + A2 * rpm ^2) $A1:$ 0	A2:	10			
				Steady State Harmonic Excitation: $Q = A *  Q  * cos(wexc't + phase)$ Excitation frequency wexc (rad/sec) = cpm $*(2\text{pi}/60)$ , and A is the Amplitude multiplier					
				rpm = excitation shaft speed, rotor speed where the excitation applied					
	Ele(Stn)	Sub	Dir	Left Amp.	Left Ana.	Right Amp.	Right Ang.	<b>Comments</b>	
2	2 2	1 1	1. 2	18385 85215	355.79 55.94	n. Ω.	0. 0		
3	8	1	$\overline{2}$	30026	2.3986	0	0		
Λ									
5									
6									
$\overline{7}$									
8									
9									
10 <sup>10</sup>									▼
<b>Insert Row</b>		Delete Row						Unit:(2) - Amp: Lbf, Phase: deg	
Tor K						Save As Save		Close	Help

**Fig. 53 – Steady State Harmonic Excitation at 40,000 cpm**

Again, the absolute displacements can be compared with the results from the base motion.





#### **\*\*\*\*\*\*\*\*\*\*\*\*\*\* Harmonic Response due to Shaft (1) Excitation \*\*\*\*\*\*\*\*\*\*\*\*\*\*\* \*\*\* Excitation Frequency = 40000. cpm \*\*\***



### **Example 5 – Turbocharger**

An automobile turbocharger is used in this demonstration. All the automobile turbochargers with floating ring bearings are operated beyond the instability threshold in linear theory and nonlinear analysis is required. Currently, in Ver 21, the Base Motion Analysis is a linear analysis for a linear system. The floating ring bearing data is save in BePerf and read directly in Rotor program. Linearized bearing coefficients are calculated first before the base motion is performed. Since this is a linear analysis, it may not give you the accurate prediction of the rotor behavior, however, it certainly give you some idea on how the rotor will behave for the base motion with the specified excitations.

# **Case 1: BaseMotion\_5a.rot**

The rotor model and some inputs are shown below:









**Fig. 54 – Turbocharger Model**

The base motion analysis is performed at a rotor speed of 125,000 rpm with excitation frequency from 5000 to 20,000 cpm.



**Fig. 54 – Base Motion Analysis**

The responses due to the base motion at some stations are shown below. To further understand the rotor behavior, a whirl speed analysis is performed at the rotor 125,000 rpm, it clearly shows that a precessional mode with a frequency of 14,401 cpm is excited by this base motion and the floating ring at the turbine end is nearly stationary and the ring at the compressor end is very active. The mode shape is also shown for reference.















